BOARD QUESTION PAPER : JULY 2023
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

(i) *All questions are compulsory.*
(ii) *There are 6 questions divided into two sections.*
(iii) *Write answers of Section-I and Section-II in the same answer book.*
(iv) *Use of logarithmic tables is allowed. Use of calculator is not allowed.*
(v) *For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.*
(vi) *Start answer to each question on a new page.*
(vii) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter eg. (a) ……../(b)……../(c)……../ (d)…….. No mark(s) shall be given if "ONLY" the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.*

Section – I

Q.1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): **(6)[12]**

i. The negation of the proposition "If 2 is prime then 3 is odd", is _____.
(a) If 2 is not prime, then 3 is not odd.
(b) 2 is prime and 3 is not odd.
(c) 2 is not prime and 3 is odd.
(d) If 2 is not prime, then 3 is odd.

ii. If A is a non-singular matrix, then det (A^{-1}) = _____.
(a) 1 (b) 0 (c) det (A) (d) $\frac{1}{\det(A)}$

iii. If $y = 2x^2 + 2^2 + a^2$ then $\frac{dy}{dx}$ = _____.
(a) $4x + 2a$ (b) $4x$ (c) $2x$ (d) $-2x$

iv. If elasticity of demand $\eta = 1$ then demand is _____.
(a) constant (b) inelastic (c) unitary elastic (d) elastic

v. $\int (1-x)^{-2}\,dx$ = _____.
(a) $(1 + x)^{-1} + c$ (b) $(1 - x)^{-1} - x + c$
(c) $(1 - x)^{-1} + c$ (d) $(1 - x)^{-1} + x + c$

vi. $\int_{-7}^{7} \frac{x^3}{x^2+7}\,dx$ = _____.
(a) 7 (b) 49 (c) $\frac{7}{2}$ (d) 0

(B) State whether the following statements are true or false (1 mark each): **(3)**

i. $p \wedge t \equiv p$

ii. $\int \frac{f'(x)}{\sqrt{f(x)}}dx = \sqrt{f(x)} + c$

iii. The integrating factor of the differential equation $\frac{dy}{dx} - y = x$ is e^{-x}.

(C) Fill in the following blanks (1 mark each): **(3)**

i. To find the value of $\int \frac{(1+\log x)}{x} dx$ the proper substitution is _____.

ii. Using definite integration area of circle $x^2 + y^2 = 25$ is _____.

iii. The order and degree of $\left(\frac{dy}{dx}\right)^3 - \frac{d^3y}{dx^3} + ye^x = 0$ are _____.

Q.2. (A) Attempt any TWO of the following questions (3 marks each): **(6)[14]**

i. Write converse, inverse and contrapositive of the implication:
"If Sanjay studies, then he will go to college".

ii. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ prove that
$A^2 - 5A + 7I = 0$, where I is 2×2 unit matrix and 0 is zero matrix of order 2.

iii. Find $\frac{dy}{dx}$, if $x^7 \cdot y^9 = (x + y)^{16}$

(B) Attempt any TWO of the following questions (4 marks each): **(8)**

i. Solve the differential equation:
$x^2 y\, dx - (x^3 + y^3)\, dy = 0$

ii. The total cost of producing x units is ₹$(x^2 + 60x + 50)$ and the price is ₹$(180 - x)$ per unit. For how many units the profit will be maximum?

iii. Evaluate: $\int \frac{x}{(x-1)^2(x+2)} dx$

Q.3. (A) Attempt any TWO of the following questions (3 marks each): **(6)[14]**

i. Using the truth table, verify
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

ii. If $x = t \cdot \log t$, $y = t^t$ then show that $\frac{dy}{dx} - y = 0$.

iii. Find the area of the region bounded by $y = x^2$ with X-axis and $x = 1$, $x = 4$.

(B) Attempt any ONE of the following questions: **(4)**

i. Solve the following equations by method of reduction:
$x + y + z = 1$, $2x + 3y + 2z = 2$, $x + y + 2z = 4$

ii. If the demand function is $D = \left(\frac{p+6}{p-3}\right)$, find the elasticity of demand at $p = 4$.

(C) Attempt any ONE of the following questions (Activity): **(4)**

i. Evaluate: $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$

Solution:

Let $I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$ …(i)

By property $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

$I = \int_1^3 \frac{\square}{\sqrt[3]{9-x} + \square} dx$ …(ii)

Adding (i) and (ii)

$$I + I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}dx + \int_1^3 \frac{\square}{\sqrt[3]{9-x}+\square}dx$$

$$= \int_1^3 \frac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}dx$$

$$2I = \int_1^3 \square\, dx$$

$$= \left[\square\right]_1^3$$

$\therefore \quad 2I = \square$

$\therefore \quad I = 1$

ii. Solve the differential equation:

$$\frac{dy}{dx} - y = 2x$$

Solution:

The differential equation $\frac{dy}{dx} - y = 2x$ is in the form of $\frac{dy}{dx} + P_y = Q$

where P = –1 and Q = 2x

$\therefore \quad$ I. F. = $e^{\int P\,dx} = \square$

$\therefore \quad$ The solution of the linear differential equation is

$\therefore \quad y \square = \int 2x \square\, dx + c$

$$= 2\left\{x\int e^{-x}dx - \int e^{-x}dx \cdot \frac{d}{dx}(x)dx\right\} + c$$

$$= 2\left\{x\int \frac{\square}{\square} - \int \frac{\square}{\square}\cdot dx\right\} + c$$

$\therefore \quad ye^{-x} = -2xe^{-x} + 2\int e^{-x}\,dx + c$

$\therefore \quad ye^{-x} = -2xe^{-x} + 2\left(\frac{e^{-x}}{-1}\right) + c$

$\therefore \quad y + \square = c\,e^x$ is the required solution of the given differential equation.

Section – II

Q.4. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): **(6)[12]**

i. The payment date after adding 3 days of grace period is known as _____.

(a) The legal due date (b) The nominal due date
(c) Days of grace (d) Date of drawing

ii. $b_{XY} \cdot b_{YX}$ = _____.

(a) $v(x)$ (b) σ_x (c) r^2 (d) $(\sigma_y)^2$

iii. If the corner points of the feasible region are (0, 10), (2, 2) and (4, 0) then the point of minimum $z = 3x + 2y$ is _____.

(a) (2, 4) (b) (0, 10) (c) (4, 0) (d) (2, 2)

iv. If job A to D have processing times 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine then the optimal sequence is _____.

(a) CDAB (b) DBCA (c) BCDA (d) ADBC

v. If E(X) = m and Var (X) = m then X follows _____.
(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) (a) and (b) both

vi. If $X \sim B\left(20, \frac{1}{10}\right)$ then Var (X) = _____.

(a) $\frac{9}{5}$ (b) 2 (c) $\frac{5}{9}$ (d) $\frac{1}{2}$

(B) State whether the following statements are true or false (1 mark each) : **(3)**

i. The amount of claim cannot exceed the amount of loss.

ii. $\frac{\sum p_0(q_0+q_1)}{\sum p_1(q_0+q_1)} \times 100$ is Marshall-Edgeworth's price index number.

iii. If r.v. X assumes values 1, 2, 3, … n with equal probabilities then E(X) = $\frac{(n+1)}{2}$.

(C) Fill in the blanks (1 mark each): **(3)**

i. The difference between the banker's discount and the true discount is called _____.

ii. A train carries at least twice as many first class passengers (y) as second class passengers (x). The constraint is given by _____.

iii. If F(x) is distribution function of discrete r.v. X with p.m.f. P(x) = $\frac{x-1}{3}$, for x = 1, 2, 3 and P(x) = 0 otherwise then F(4) = _____.

Q.5. (A) Attempt any TWO of the following questions (3 marks each): **(6)[14]**

i. A lady plans to save for her daughter's marriage. She wishes to accumulate a sum of ₹ 4,64,100 at the end of 4 years. What amount should she invest every year if she gets an interest of 10% p.a. compounded annually?
[Given : $(1.1)^4 = 1.4641$].

ii. The following table shows the production of gasoline in U.S.A. for the years 1962 to 1976:

Year	Production (million Barrels)	Year	Production (million Barrels)
1962	0	1970	6
1963	0	1971	7
1964	1	1972	8
1965	1	1973	9
1966	2	1974	8
1967	3	1975	9
1968	4	1976	10
1969	5		

Obtain trend values for the above data using 5 yearly moving averages.

iii. Find x if the price index number by simple aggregate method is 125:

Commodity	P	Q	R	S	T
Base Year Price (in ₹)	8	12	16	22	18
Current Year Price (in ₹)	12	18	x	28	22

(B) Attempt any TWO of the following questions (4 marks each) : **(8)**

i. Minimize: $z = 7x + y$
Subject to $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$.

ii. A bill of a certain sum drawn on 28th February, 2007 for 8 months was encashed on 26th March, 2007 for ₹ 10,992 at 14% p.a. Find the face value of the bill.

iii. There are five jobs, each of which must go through two machines in the order XY. Processing times (in hours) are given below. Determine the optimal sequence for the jobs that will minimize the total elapsed time. Also find the total elapsed time and idle time for machine Y.

Job	A	B	C	D	E
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

Q.6. (A) Attempt any TWO of the following questions (3 marks each) : (6)[14]

i. The two regression equations are $5x - 6y + 90 = 0$ and $15x - 8y - 130 = 0$. Find $\bar{x}, \bar{y}$, r.

ii. If $\Sigma p_0q_0 = 140$, $\Sigma p_0q_1 = 200$, $\Sigma p_1q_0 = 350$ and $\Sigma p_1q_1 = 460$, find Laspeyre's, Paasche's, Dorbish-Bowley's price index numbers.

iii. Find expected value and variance of X using the following p.m.f.

x	–2	–1	0	1	2
P(x)	0.2	0.3	0.1	0.15	0.25

(B) Attempt any ONE of the following questions (4 marks each) : (4)

i. The following results were obtained from records of age (X) and systolic blood pressure (Y) of a group of 10 men:

	X	Y
Mean	50	140
Variance	150	165

and $\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = 1120$. Find the prediction of blood pressure of a man of age 40 years.

ii. Following table shows the number of traffic fatalities (in a state) resulting from drunken driving for years 1975 to 1983 :

Years	1975	1976	1977	1978	1979	1980	1981	1982	1983
No. of deaths	0	6	3	8	2	9	4	5	10

Fit a trend line to the above data by the method of least squares.

(C) Attempt any ONE of the following questions (Activity) (4 marks each) : (4)

i. A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesmen in each district is as below:

Salesmen	District			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

Find the assignment of salesmen to various districts which will yield maximum profit.

Solution: It is a maximization problem. Subtract all the elements from $\boxed{\quad}$.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 6 & 4 & 5 \\ 4 & 3 & 1 & 1 \\ 1 & 1 & 5 & 2 \\ 3 & 2 & 2 & 1 \end{bmatrix} \end{array}$$

∴ Subtract the smallest element of each row from the elements of that row:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 6 & 4 & 5 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

∴ Subtract the smallest element of each column from the elements of that column:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 6 & 4 & 5 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

∴ Since the number of lines covering zeros is equal to the order of matrix, the optimal solution has reached:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 6 & 4 & 5 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The optimal solution is obtained:

Salesmen	Districts	Profits (₹)
A	1	16
B		
C		
D	4	15

∴ Total profit = ₹ ☐

ii. If X has Poisson distribution with parameter m and P[X = 2] = P[X = 3], then find P[X ≥ 2].
[Given : $e^{-3} = 0.0497$]
Solution :
X ~ P(m)

∴ $P[X = x] = \frac{e^{-m}m^x}{x!}$

∴ $P[X = 2] = \frac{e^{-m}m^2}{2!}$

∴ P[X = 3] = ☐

Now P[X = 2] = P[X = 3]

∴ $\frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!}$

∴ m = ☐

Now, P[X ≥ 2] = 1 – ☐

$= 1 - [P(X = 0) + P(X = 1)]$

$= 1 - \left[\frac{e^{-3} \cdot 3^0}{0!} + \frac{e^{-3} \cdot 3^1}{1!}\right]$

$= 1 - e^{-3}[1+3]$

$= 1 - 0.0497 \times 4$

Hence P[X ≥ 2] is ☐