



BOARD QUESTION PAPER: JULY 2022

MATHEMATICS AND STATISTICS

Time: 3 Hrs.

Max. Marks: 80

General Instructions:

- (1) All questions are compulsory.
- (2) There are 6 questions divided into two sections.
- (3) Write answers of Section-I and Section-II in the same answer book.
- (4) Use of logarithmic tables is allowed. Use of calculator is not allowed.
- (5) For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.
- (6) Start answer to each question on a new page.
- (7) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter eg. (a)/(b)/(c)/(d) No mark(s) shall be given if “ONLY” the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.

SECTION – I

Q.1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each):

(6)[12]

- (i) if p: He is intelligent
q: He is strong.
are statements in logic. Then symbolic form of statement.
'It is false that, he is intelligent or strong' is _____.
(a) $\sim p \vee \sim q$ (b) $\sim (p \wedge q)$ (c) $\sim (p \vee q)$ (d) $p \vee \sim q$
- (ii) If $y = e^{\log x}$ then $\frac{dy}{dx} =$ _____.
(a) $\frac{e^{\log x}}{x}$ (b) $\frac{1}{x}$ (c) 0 (d) $\frac{1}{2}$
- (iii) If $x = 2at^2, y = 4at$ then $\frac{dy}{dx} =$ _____.
(a) $\frac{1}{t}$ (b) $\frac{-1}{2at^2}$ (c) $\frac{1}{2at^3}$ (d) $\frac{1}{4at^3}$
- (iv) If elasticity of demand $\eta = 1$ then demand is _____.
(a) constant (b) inelastic (c) unitary elastic (d) elastic
- (v) The statement $p \rightarrow q$ is equivalent to _____.
(a) $p \rightarrow \sim q$ (b) $\sim p \vee q$ (c) $\sim p \rightarrow \sim q$ (d) $p \vee \sim q$
- (vi) The matrix $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ is _____.
(a) identity matrix (b) scalar matrix
(c) null matrix (d) column matrix

(B) State whether the following statements are true or false (1 mark each):

(3)

- (i) The area of the portion lying above the X-axis is positive.
- (ii) The order of highest derivative occurring in the differential equation is called degree of the differential equation.
- (iii) If A and B are conformable for the product AB then $(AB)^T = A^T B^T$.



(C) Fill in the following blanks (1 mark each):

(3)

- (i) If $y = x \cdot \log x$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
- (ii) If $\int_0^a 3x^2 dx = 8$, then $a = \underline{\hspace{2cm}}$.
- (iii) Order and degree of differential equation are always $\underline{\hspace{2cm}}$ integers.

Q.2. (A) Attempt any TWO of the following (3 marks each):

(6)[14]

- (i) Write the converse, inverse and contrapositive of the statement, "If he studies, then he will go to college".
- (ii) Find x and y if $\left\{ 4 \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$.
- (iii) Find the area of the region bounded by the curve $y = x^2 + 1$, lines $x = 0$, $x = 3$ and X-axis.

(B) Attempt any TWO of the following (4 marks each):

(8)

- (i) If $x^3 y^3 = x^2 - y^2$, then find $\frac{dy}{dx}$.
- (ii) Find MPC, MPS, APC and APS, if the expenditure E_e of a person with income I is given as $E_e = (0.0003) I^2 + (0.075) I$ when $I = 1000$
- (iii) Find the population of a city at any time 't' given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 30,000 to 40,000.

Q.3. (A) Attempt any TWO of the following questions (3 marks each):

(6)[14]

- (i) Using the truth table, examine whether the given statement pattern is tautology, contradiction or contingency.
 $p \rightarrow (q \vee r)$
- (ii) Find the inverse of matrix $\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$ by adjoint method.
- (iii) Evaluate: $\int \frac{1}{x^2 + 4x - 5} dx$

(B) Attempt any ONE of the following (4 marks each):

(4)

- (i) Evaluate: $\int x^3 e^{x^2} dx$
- (ii) Evaluate: $\int_2^7 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx$

(C) Attempt any ONE of the following questions (Activity):

(4)

- (i) A metal wire of 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum by completing following activity:

Solution:

Let x and y be the length breadth of rectangle, then

$$x + y = \boxed{\hspace{2cm}}$$

then area of rectangle $= 18x - x^2$

Let $f(x)$ be the area of rectangle

$$f(x) = 18x - x^2$$

$$f'(x) = \boxed{\hspace{2cm}}$$

$$f''(x) = -2$$



For extreme value $f'(x) = 0$

$$x = \boxed{}$$

Now $f''(q) < 0$

$\therefore f(x)$ [Area] is maximum when dimensions (length and breadth are) $\boxed{}$

- (ii) Solve the following differential equation $x^2 y \, dx - (x^3 + y^3) \, dy = 0$ by filling the boxes in the following activity:

Solution:

$$x^2 y \, dx - (x^3 + y^3) \, dy = 0$$

This is homogeneous equation

$\therefore$ we put $y = vx$

$$\therefore \frac{dy}{dx} = \boxed{}$$

From given equation

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 \cdot xv}{x^3 + v^3 x^3}$$

$$= \frac{v}{1 + v^3}$$

$$x \cdot \frac{dv}{dx} = \frac{v}{v^3 + 1} - v$$

$$= \frac{v - v^4 - v}{v^3 + 1}$$

$$\therefore x \frac{dv}{dx} = \frac{-v^4}{v^3 + 1}$$

$$\therefore \boxed{} \, dv = \frac{-dx}{x}$$

Integrating both sides

$$\int \left(\frac{1}{v} + \frac{1}{v^4} \right) dv = - \int \frac{dx}{x}$$

$$\boxed{} + \log c = -\log x$$

$$\log cvx = \frac{1}{3v^3}$$

Substituting the value of v we get

$\boxed{}$ is the general solution.

SECTION – II

Q.4. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each):

(6)[12]

- (i) An agent who gives guarantee to his principal that the party will pay the sale price of goods is called _____.
 (a) Auctioneer (b) Delcredere agent (c) Factor (d) Broker
- (ii) X is a number obtained on upper most face when a fair die is thrown then $E(x) = \underline{\hspace{2cm}}$.
 (a) 3 (b) 3.5 (c) 4 (d) 4.5



(iii) The cost of living Index number using Aggregate Expenditure method is given by _____.

(a) $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

(b) $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$

(c) $\frac{\sum p_1 q_1}{\sum p_1 q_0} \times 100$

(d) $\frac{\sum p_0 q_0}{\sum p_1 q_0} \times 100$

(iv) In sequencing an optimal path minimizes _____.

(a) Elapsed time

(b) Idle time

(c) Both (a) and (b)

(d) Ready time

(v) The marked price is also called as _____.

(a) Cost price (b) Selling price

(c) List price

(d) Invoice price

(vi) If $X \sim B\left(20, \frac{1}{10}\right)$, then $E(x) =$ _____.

(a) 2

(b) 5

(c) 4

(d) 3

(B) State whether the following statements are true or false (1 mark each):

(3)

(i) In an assignment problem, if number of columns is greater than number of rows then a dummy column is added.

(ii) The optimum value of the objective function of LPP occurs at the centre of the feasible solution.

(iii) Regression equation of X on Y is $y - \bar{y} = b_{yx}(x - \bar{x})$.

(C) Fill in the blanks (1 mark each):

(3)

(i) If X is discrete random variable which takes the values $x_1, x_2, \dots, x_n$ then $\sum_{i=1}^n p(x_i) =$ _____.

(ii) The constraint that a factory has to employ more women (y) than men (x) is given by _____.

(iii) If F(x) is distribution function of discrete r.v. X with p.m.f.

$$p(x) = \frac{x-1}{3}, \text{ for } x = 1, 2, 3$$

$$= 0, \text{ otherwise}$$

$$\text{then } F(4) = \text{_____}.$$

Q.5. (A) Attempt any TWO of the following questions (3 marks each):

(6)[14]

(i) Using 1995 as base year in the following problem; find the price index number by aggregate method.

Commodity	A	B	C	D	E
Price (in ₹) in 1995	42	30	54	70	120
Price (in ₹) in 2005	60	55	74	110	140

(ii) Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost matrix is given below:

Machines	Places				
	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	—	5	4
M_3	—	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.



- (iii) The following table shows the production of gasoline in U.S.A. for the years 1962 to 1976. Obtain trend values using 5-yearly moving averages:

Year	Production (Million Barrels)	Year	Production (Million Barrels)
1962	0	1970	6
1963	0	1971	7
1964	1	1972	8
1965	1	1973	9
1966	2	1974	8
1967	3	1975	9
1968	4	1976	10
1969	5		

(B) Attempt any TWO of the following questions (4 marks each):

(8)

- (i) There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?
- (ii) Minimize $z = 4x + 2y$
 Subject to $3x + y \geq 27$
 $x + y \geq 21$
 $x \geq 0, y \geq 0$

Solve above LPP by graphical method.

- (iii) A building is insured for 75% of its value. The annual premium at 0.70 percent amounts to ₹ 2,625. If the building is damaged to the extent of 60% due to fire, how much can be claimed under policy?

Q.6. (A) Attempt any TWO of the following questions (3 marks each):

(6)[14]

- (i) If $\sum p_0 q_0 = 140$, $\sum p_0 q_1 = 200$, $\sum p_1 q_0 = 350$, $\sum p_1 q_1 = 460$, find Laspeyre's price index number, Pasche's price index number and Dorbish-Bowley's price Index number.
- (ii) For a bivariate data $\bar{x} = 53$, $\bar{y} = 28$, $b_{YX} = -1.2$; $b_{XY} = -0.3$, find
 (a) Correlation coefficient between X and Y.
 (b) Estimate Y for X = 50
- (iii) Defects on plywood sheet occur at random with the average of one defect per 50 sq.ft. Find the probability that such a sheet has (i) no defect (ii) at least one defect
 [use $e^{-1} = 0.3678$]

(B) Attempt any ONE of the following questions (4 marks each):

(4)

- (i) Find the sequence that minimizes the total elapsed time to complete the following jobs. Each job is processed in order AB.

	Jobs (Processing time in minutes)						
	I	II	III	IV	V	VI	VII
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Find the total elapsed time and the idle times for both the machines.

- (ii) From the following data, find the regression equation of Y on X and estimate Y when X = 10

X	1	2	3	4	5	6
Y	2	4	7	6	5	6



(C) Attempt any ONE of the following questions (Activity):

(4)

- (i) A bill was drawn on 14th April for ₹ 7,000 and was discounted on 6th July at 5% p.a. The banker paid ₹ 6,930 for the bill. Find the period of the bill by filling boxes in the following activity.

Solution:

$$SD = ₹ 7,000, r = 5\%, CV = 6,930$$

$$BD = \boxed{}$$

$$BD = \frac{SD \times n \times r}{100}$$

$$n = \boxed{} \text{ days}$$

Number of days to be counted from date of discounting i.e., 6th July

July	Aug.	Sep.	Total

∴ Period of Bill is $\boxed{}$ month

- (ii) Find the equation of trend line for the following data showing production of steel (in millions of tonnes) for years 1976 to 1986 by filling boxes in the following activity:

Year	1976	1977	1978	1979	1980	
Production	0	4	4	2	6	
Year	1981	1982	1983	1984	1985	1986
Production	8	5	9	4	10	10

Solution:

Year (t)	Production y_t	$u = t - 1981$	u^2	uy_t
1976	0	-5	25	0
1977	4	-4	16	-16
1978	4	-3	9	-12
1979	2	-2	4	-4
1980	6	-1	1	-6
1981	8	0	0	0
1982	5	1	1	5
1983	9	2	4	18
1984	4	3	9	12
1985	10	4	16	40
1986	10	5	25	50
Total	$\sum y_t = 62$	$\sum u = 0$	$\sum u^2 = 110$	$\sum uy_t = 87$

$$u = t - 1981$$

Let equation of trend line be

$$y_t = \boxed{} \dots(1)$$

$$\sum y_t = na' + b'\sum u \dots(2)$$

$$\sum uy_t = a'\sum u + b'\sum u^2 \dots(3)$$

$$a' = \boxed{}$$

$$b' = \boxed{}$$

∴ Equation of trend line is $y_t = \boxed{}$