BOARD QUESTION PAPER : JULY 2023

MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

*The question paper is divided into **FOUR** sections.*

*(1) **Section A:** Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
*Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.*

*(2) **Section B:** Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

*(3) **Section C:** Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

*(4) **Section D:** Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a)…… / (b)…….. /(c)…….. /(d)…….., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.

(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

i. The dual of statement $p \wedge \sim q$ is equivalent to ___________.

(a) $\sim p \wedge q$ (b) $p \leftrightarrow q$ (c) $\sim p \vee q$ (d) $\sim p \rightarrow \sim q$ (2)

ii. If $|\bar{a}| = 3$, $|\bar{b}| = 4$, then a value of λ for which $a + \lambda\bar{b}$ is perpendicular to $\bar{a} - \lambda\bar{b}$ is _______.

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$ (2)

iii. The acute angle between the lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$ is _______.

(a) 60° (b) 30° (c) 45° (d) 90° (2)

iv. The vector equation of the plane passing through point $A(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is _______.

(a) $(\bar{r} - \bar{a}) \times (\bar{b} \times \bar{c}) = \bar{0}$ (b) $(\bar{r} - \bar{a}) \cdot (\bar{b} + \bar{c}) = 0$

(c) $(\bar{r} - \bar{a}) \cdot (\bar{b} \times \bar{c}) = 0$ (d) $(\bar{r} - \bar{a}) \times (\bar{b} - \bar{c}) = \bar{0}$ (2)

v. If $x = at^4$, $y = 2at^2$, then $\frac{dy}{dx} =$ _______.

(a) $\frac{1}{t^2}$ (b) t^2 (c) $2t^2$ (d) $-\frac{1}{t^2}$ (2)

vi. The area bounded by the curve $y = 2x$, the Y-axis, the X-axis and $x = 3$ is _______.

(a) 3 sq. units (b) 6 sq. units (c) 9 sq. units (d) 12 sq. units (2)

vii. The differential equation $y\frac{dy}{dx} + x = 0$ represents family of _______.

(a) circle (b) parabola (c) ellipse (d) hyperbola (2)

viii. If $f(x) = kx^2(1 - x)$, for $0 < x < 1$
$= 0$, otherwise, is the probability distribution function of a random variable X, then the value of k is _______.
(a) 12 (b) 10 (c) –9 (d) –12 (2)

Q.2. Answer the following questions: **[4]**

i. Write the domain of inverse secant function. (1)

ii. Find the value of k, if lines represented by $kx^2 + 4xy - 4y^2 = 0$ are perpendicular to each other. (1)

iii. Evaluate: $\int \frac{1}{x\sqrt{\log x}} dx$ (1)

iv. Obtain the differential equation by eliminating the arbitrary constant from the equation $y^2 = 4ax$. (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Write the following compound statements symbolically:
(i) Nagpur is in Maharashtra and Chennai is in Tamilnadu.
(ii) If ΔABC is right angled at B, then $m\angle A + m\angle C = 90°$ (2)

Q.4. Find the co-factors of the elements a_{11} and a_{21} of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. (2)

Q.5. Find the solution of $\cos\theta = \frac{1}{2}$, where $0 \le \theta < 2\pi$. (2)

Q.6. Find the joint equation of lines passing through the origin having slopes, 2 and 3. (2)

Q.7. If the vectors $-3\hat{i} + 4\hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{k}$ and $\hat{i} - p\hat{j}$ are coplanar, then find the value of p. (2)

Q.8. Find the direction cosines of the vector $2\hat{i} + 2\hat{j} - \hat{k}$. (2)

Q.9. Find the equation of tangent to the curve $y = x^2 + 2e^x + 2$ at the point (0, 4). (2)

Q.10. Evaluate: $\int \frac{3^x - 4^x}{5^x} dx$ (2)

Q.11. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx$ (2)

Q.12. Find the area of the region bounded by the curve $y = x^2$, the X-axis and the lines $x = 1$, $x = 3$. (2)

Q.13. Find the integrating factor (I. F.) of the differential equation:
$\frac{dy}{dx} + y = e^{-x}$ (2)

Q.14. Given that X ~ B (n, p).
If p = 0.6 and E(X) = 6, find n and Var(x). (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$ (3)

Q.16. If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects an angle between the co-ordinate axes then show that $(a + b)^2 = 4h^2$ (3)

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Q.17. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be the third point on the line AB dividing the segment AB externally in the ratio m : n. Then prove that $\bar{r} = \dfrac{m\bar{b} - n\bar{a}}{m - n}$ (3)

Q.18. Find the position vector of point P such that OP is inclined to X – axis at 45° and to Y – axis at 60° and OP = 12 units. (3)

Q.19. Find the vector equation of the line passing through the point having position vector $2\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$. (3)

Q.20. The foot of perpendicular drawn from the origin to a plane is M(1, 2, 0). Find the vector equation of the plane. (3)

Q.21. If $\log_{10}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = 2$, show that $\dfrac{dy}{dx} = \dfrac{-99x^2}{101y^2}$ (3)

Q.22. A wire of length 36 meters is bent in the form of rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q.23. Evaluate: $\displaystyle\int \frac{e^x}{1 + e^{-x}}\, dx$ (3)

Q.24. Solve the differential equation:

$\dfrac{dy}{dx} = (4x + y + 1)^2$ (3)

Q.25. The probability distribution of X is as follows:

X	0	1	2	3	4
$P(X = x)$	0.1	K	2K	2K	K

Find: (i) K (ii) $P(X < 2)$ (iii) $P(X \geq 3)$ (3)

Q.26. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg. (3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Construct the truth table for the statement pattern $(p \rightarrow q) \wedge [(q \rightarrow r) \rightarrow (p \rightarrow r)]$ and interpret your result. (4)

Q.28. Solve the following equations by using method of inversion:

$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

Q.29. Prove that, in ΔABC,

$\tan\left(\dfrac{A - B}{2}\right) = \left(\dfrac{a - b}{a + b}\right) \cot\left(\dfrac{c}{2}\right)$ (4)

Q.30. Solve the linear programming problem by graphical method:

Maximize: $z = 3x + 5y$

Subject to: $x + 4y \leq 24$

$3x + y \leq 21$

$x + y \leq 9$ and $x \geq 0, y \geq 0$

Also find maximum value of z. (4)

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Q.31. If $y = \mathrm{f}(x)$ is a differentiable function of x on an interval 1 and y is one-one, onto and $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$ on 1,

then prove that $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{\frac{\mathrm{d}y}{\mathrm{d}x}}$.

where $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$. Hence prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot^{-1}x) = \frac{-1}{1+x^2}$ (4)

Q.32. The volume of the spherical ball is increasing at the rate of 4π cc/sec. Find the rate at which the radius and the surface area are changing when the volume is 288π cc. (4)

Q.33. Prove that: $\int \frac{1}{x^2 - \mathrm{a}^2}\,\mathrm{d}x = \frac{1}{2\mathrm{a}} \log\left|\frac{x-\mathrm{a}}{x+\mathrm{a}}\right| + \mathrm{c}$

Hence evaluate: $\int \frac{1}{x^2 - 3}\,\mathrm{d}x$ (4)

Q.34. Evaluate: $\int_0^{\frac{\pi}{2}} \cos^3 x\,\mathrm{d}x$ (4)

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